# Sample Question Paper - 26
# Mathematics-Standard (041)
# Class- X, Session: 2021-22
# TERM II

*Time Allowed : 2 hours* *Maximum Marks : 40*

## *General Instructions :*

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

## SECTION - A

**1.** In a class test, 50 students obtained marks are as follows. Find the modal class and the median class.

| **Marks** | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|---|---|---|---|---|---|
| **Number** | 4 | 6 | 25 | 10 | 5 |

**2.** The sum of the first $n$ terms of an A.P. is $4n^2 + 2n$. Find the $n^{\text{th}}$ term of this A.P.

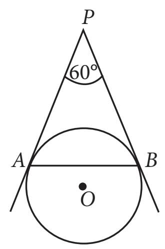
**OR**

The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference of the A.P.

**3.** The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the surface area of the cube.

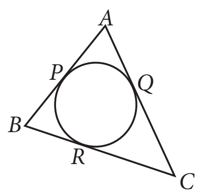
**4.** The top of two poles are connected by a 50 m wire which makes an angle of 60° with the horizontal, then find the distance (in m) between the two poles.

**5.** In the given figure, $AP$ and $BP$ are tangents to a circle with centre $O$, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord $AB$.

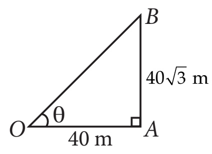



**OR**

In the given figure, if $AP = PB$, then show that $AC = BC$.



6. Find the angle of elevation of the top of a pole of height $40\sqrt{3}$ m from a point 40 m away from its foot.




## SECTION - B

7. Draw a circle of radius 1.8 cm. Take a point *P* on it. Draw a tangent to the circle at the point *P*, without using its centre.

8. Consider the following table:

| **Class interval** | 10-14 | 14-18 | 18-22 | 22-26 | 26-30 |
|---|---|---|---|---|---|
| **Frequency** | 5 | 11 | 16 | 25 | 19 |

Find the mode of the above data.

**OR**

Find the mean of the following data.

| **Class- interval** | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| **Frequency** | 3 | 5 | 9 | 5 | 3 |

9. From the top of a 60 m high building, the angles of depression of the top and the bottom of a tower are 45° and 60° respectively. Find the height of the tower. $\left[\text{Take } \sqrt{3} = 1.73\right]$

10. If the median of the following distribution is 46, find the missing frequencies *p* and *q*.

| **Class-interval** | **Frequency** |
|---|---|
| 10-20 | 12 |
| 20-30 | 30 |
| 30-40 | *p* |
| 40-50 | 65 |
| 50-60 | *q* |
| 60-70 | 25 |
| 70-80 | 18 |
| Total | 230 |

## SECTION - C

11. Find the values of *k* for which the quadratic equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has equal roots. Also find the roots.

**OR**

Adil and Aarav together have 45 silver coins. Both of them lost 5 silver coins each and the product of the number of silver coins they now have, is 124. Find out how many silver coins they had to start with?

12. If the sum of first 10 terms of an A.P. is 140 and the sum of first 16 terms is 320, then find the A.P. and hence find the sum of first *m* terms.

## Case Study - 1

**13.** Smita always finds it confusing with the concept of tangent of a circle. But this time she has determined herself to get concepts easier. So, she started listing down about the concept of tangent of a circle. Here, some points in question form are listed by Smita in her notes. Try answering them to clear your concepts also.

(i) Find the length of the tangent drawn from a point 8 cm away from the centre of a circle of radius 6 cm.

(ii) If the angle between two radii of a circle is 140°, then find the angle between the tangents at the ends of the radii.

## Case Study - 2

**14.** Meera and Dhara have 12 and 8 coins respectively each of radius 3.5 cm and thickness 0.5 cm. They place their coins one above the other to form solid cylinders.

Based on the above information, answer the following questions.

(i) Find the ratio of curved surface area of the cylinders made by Meera and Dhara.

(ii) Find the ratio of the volume of the cylinders made by Meera and Dhara.

**Solution**

**MATHEMATICS STANDARD 041**

**Class 10 - Mathematics**

**1.** The frequency distribution table for the given data can be drawn as :

| Marks | Frequency ($f_i$) | Cumulative frequency ($c.f.$) |
|---|---|---|
| 0-20 | 4 | 4 |
| 20-40 | 6 | 10 |
| 40-60 | 25 | 35 |
| 60-80 | 10 | 45 |
| 80-100 | 5 | 50 |
| | $\Sigma f_i = 50$ | |

$\therefore$ 40-60 is the modal class as it has highest frequency.

Also, $\frac{N}{2} = \frac{50}{2} = 25$

The $c.f.$ just greater than 25 lies in the interval 40-60.
Hence the median class is 40-60.

**2.** We have, $S_n = 4n^2 + 2n$

$\therefore$ $S_{n-1} = 4(n-1)^2 + 2(n-1)$
$= 4(n^2 + 1 - 2n) + 2n - 2$
$= 4n^2 + 4 - 8n + 2n - 2 = 4n^2 - 6n + 2$

Now, $n^{th}$ term of the A.P., $a_n = S_n - S_{n-1}$
$= (4n^2 + 2n) - (4n^2 - 6n + 2) = 8n - 2$

**OR**

Let the first term be $a$ and $d$ be the common difference of the A.P.

Given, $a_5 = 20 \Rightarrow a + 4d = 20$ ...(i)

Also, $a_7 + a_{11} = 64$

$\Rightarrow a + 6d + a + 10d = 64 \Rightarrow 2a + 16d = 64$

$\Rightarrow a + 8d = 32$ ...(ii)

Subtracting (i) from (ii), we have

$4d = 12 \Rightarrow d = 3$

**3.** Volume of given cuboid $= 100 \times 80 \times 64$
$= 512000 \text{ cm}^3$

Now, cuboid is melted and recast into a cube.
Let side of the cube = $a$ cm
Also, volume of the cube = volume of the cuboid

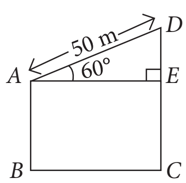
$\Rightarrow a^3 = 512000 \Rightarrow a = 80$

$\therefore$ Surface area of cube $= 6a^2 = 6 \times (80)^2 = 38400 \text{ cm}^2$

**4.** Let $AB$ and $CD$ are the two poles and length of the wire attached to their tops *i.e.*, $AD = 50$ m.

Now, draw $AE \perp CD$.

In right $\Delta ADE$,




$\cos 60° = \frac{AE}{AD}$

$\Rightarrow \frac{1}{2} = \frac{AE}{50}$

$\Rightarrow AE = \frac{50}{2} = 25$ m

Hence, distance between two poles is 25 m.

**5.** Given, $PA$ and $PB$ are tangents from an external point $P$.

$\therefore$ $PA = PB = 5$ cm

$\Rightarrow \angle PAB = \angle ABP$ ($\because$ Angles opposite to equal sides are equal)

In $\Delta APB$, by angle sum property

$\angle APB = \angle PAB = \angle ABP = 60°$

$\Rightarrow$ $\Delta PAB$ is an equilateral triangle.

$\therefore$ $AB = PB = PA = 5$ cm

**OR**

Since, tangents drawn from an external point of a circle are equal.

$\therefore$ $AP = AQ$ ; ...(i)

$BP = BR$ ; ...(ii)

$CR = CQ$ ...(iii)

Adding (i) and (iii), $AP + CR = AQ + CQ$

$\Rightarrow PB + CR = AC$ [$\because$ Given that $AP = PB$]

$\Rightarrow BR + CR = AC$ [Using (ii)]

$\Rightarrow BC = AC$

**6.** Let $AB$ be the pole and $O$ be the point of observation 40 m away from the foot of the pole $AB$.

Then, $AB = 40\sqrt{3}$ m and $OA = 40$ m

Let $\theta$ be the angle of elevation of the top of the pole.

*i.e.*, $\angle BOA = \theta$

In right $\Delta OAB$, $\tan\theta = \frac{AB}{OA} = \frac{40\sqrt{3}}{40} = \sqrt{3}$

But we know that, $\tan 60° = \sqrt{3}$

$\Rightarrow \tan\theta = \tan 60° \Rightarrow \theta = 60°$

So, required angle of elevation is 60°.

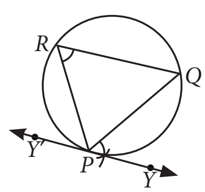
**7. Steps of construction :**

**Step-I :** Draw a circle of radius 1.8 cm and take a point $P$ on the circle.

**Step-II :** Draw a chord $PQ$ through the point $P$ on the circle.

**Step-III :** Take any point $R$ in the major arc and join $PR$ and $RQ$.

**Step-IV :** On taking $PQ$ as base, construct $\angle QPY = \angle PRQ$.



**Step-V :** Produce $YP$ to $Y'$.

Then, $Y'PY$ is the required tangent at point $P$.

**8.** Here, the maximum frequency is 25 and the corresponding modal class is 22–26.

$\therefore$ $l = 22, h = 4, f_1 = 25, f_0 = 16$ and $f_2 = 19$

$$\therefore \text{ Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 22 + \left(\frac{25-16}{50-16-19}\right) \times 4 = 22 + \frac{9}{15} \times 4$$

$$= 22 + \frac{12}{5} = 24.4$$

**OR**

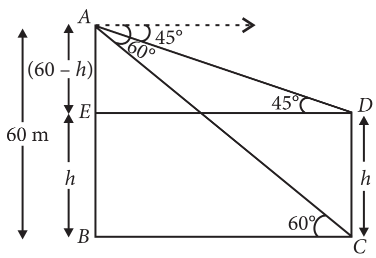
Let the assumed mean $a = 25$.

Now, the frequency distribution table from the given data can be drawn as:

| Class-interval | Class mark ($x_i$) | Frequency ($f_i$) | $d_i = x_i - a$ | $f_i d_i$ |
|---|---|---|---|---|
| 0-10 | 5 | 3 | –20 | –60 |
| 10-20 | 15 | 5 | –10 | –50 |
| 20-30 | 25 | 9 | 0 | 0 |
| 30-40 | 35 | 5 | 10 | 50 |
| 40-50 | 45 | 3 | 20 | 60 |
| | | $\Sigma f_i = 25$ | | $\Sigma f_i d_i = 0$ |

$$\therefore \text{ Mean, } \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 25 + \frac{0}{25} = 25$$

**9.** Let $AB$ be the building and $DC$ be the tower of height $h$ m.




$$\text{In } \Delta ABC, \ \tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{BC} \Rightarrow BC = \frac{60}{\sqrt{3}} \text{ m}$$

$$\text{In } \Delta AED, \ \tan 45° = \frac{AE}{ED}$$

$$\Rightarrow 1 = \frac{(60-h)}{\frac{60}{\sqrt{3}}} \quad (\because BC = ED)$$

$$\Rightarrow \frac{60}{\sqrt{3}} = 60 - h \Rightarrow h = 60 - \frac{60}{\sqrt{3}} = 60\frac{(\sqrt{3}-1)}{\sqrt{3}}$$

$$\Rightarrow h = 60\left(\frac{1.73-1}{1.73}\right) = 60\left(\frac{0.73}{1.73}\right) = 25.31$$

$\therefore$ Height of tower = 25.31 m

**10.**

| Class-interval | Frequency ($f_i$) | Cumulative frequency ($c.f.$) |
|---|---|---|
| 10-20 | 12 | 12 |
| 20-30 | 30 | 42 |
| 30-40 | $p$ | $42 + p$ |
| 40-50 | 65 | $107 + p$ |
| 50-60 | $q$ | $107 + p + q$ |
| 60-70 | 25 | $132 + p + q$ |
| 70-80 | 18 | $150 + p + q$ |
| Total | $\Sigma f_i = 230$ | |

Since, median = 46, which lies in the interval 40-50, therefore its median class is 40-50.

So, $l = 40$, $n = 230$, $c.f. = 42 + p$, $f = 65$, $h = 10$

$$\text{Now, median} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$\therefore \ 46 = 40 + \left(\frac{\frac{230}{2} - (42+p)}{65}\right) \times 10$$

$$\Rightarrow 6 = \frac{(115-42-p)}{65} \times 10$$

$\Rightarrow 39 = 73 - p \Rightarrow p = 34$ ...(i)

Also, $150 + p + q = 230$

$\Rightarrow 150 + 34 + q = 230$ (Using (i))

$\Rightarrow q = 230 - 184 \Rightarrow q = 46$

$\therefore$ Missing frequencies are 34 and 46.

**11.** We have, $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ ...(1)

Since roots are equal. $\therefore D = 0$

$\Rightarrow [2(k + 1)]^2 - 4(3k + 1) = 0$

$\Rightarrow [4(k^2 + 1 + 2k)] - 4(3k + 1) = 0$

$\Rightarrow k^2 + 2k + 1 - 3k - 1 = 0$

$\Rightarrow k^2 - k = 0 \Rightarrow k(k - 1) = 0 \Rightarrow k = 0$ or $k = 1$

When $k = 0$, (1) becomes $x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0 \Rightarrow x = -1, -1$

When $k = 1$, (1) becomes $4x^2 + 4x + 1 = 0$

$$\Rightarrow (2x + 1)^2 = 0 \Rightarrow x = \frac{-1}{2}, \frac{-1}{2}$$

Thus, equal roots of given equation are either $-1$ or $\frac{-1}{2}$.

**OR**

Given, Adil and Aarav together have 45 silver coins.

Let Adil has $x$ silver coins.

Then, number of silver coins Aarav had = 45 – $x$

∵ Both of them lost 5 silver coins each.

∴ The number of silver coins Adil had = $x$ – 5

and the number of silver coins Aarav had = 45 – $x$ – 5 = 40 – $x$

Now, product of the number of silver coins = 124 [Given]

∴ $(x - 5)(40 - x) = 124$

⇒ $40x - x^2 - 200 + 5x = 124$

⇒ $-x^2 + 45x - 200 - 124 = 0$

⇒ $x^2 - 45x + 324 = 0$, [Multiplying by (–1)]

which is the required quadratic equation.

Now, by factorisation method, we get

$x^2 - 36x - 9x + 324 = 0$

⇒ $x(x - 36) - 9(x - 36) = 0 \Rightarrow (x - 36)(x - 9)$

⇒ $x - 36 = 0$ or $x - 9 = 0 \Rightarrow x = 36$ or $x = 9$

When Adil has 36 silver coins, then Aarav has 45 – 36 = 9 silver coins

When Adil has 9 silver coins, then Aarav has 45 – 9 = 36 silver coins.

**12.** Let $a$ be the first term and $d$ be the common difference of the A.P.

Given, sum of first 10 terms, $S_{10} = 140$

⇒ $\frac{10}{2}[2a+(10-1)d] = 140 \Rightarrow 2a+9d = \frac{140}{5}$

⇒ $2a + 9d = 28$ ...(i)

Also, given sum of first 16 terms, $S_{16} = 320$

⇒ $\frac{16}{2}[2a+(16-1)d] = 320$

⇒ $2a + 15d = \frac{320}{8} \Rightarrow 2a+15d = 40$ ...(ii)

On subtracting (i) from (ii), we get $6d = 12 \Rightarrow d = 2$

Putting $d = 2$ in (i), we get

$2a + 9(2) = 28$

⇒ $2a = 28 - 18 \Rightarrow a = 10/2 = 5$

Thus, $a = 5$ and $d = 2$

∴ A.P. will be 5, 5 + 2, 5 + 2(2), 5 + 3(2),.....

*i.e.*, 5, 7, 9, 11, ...

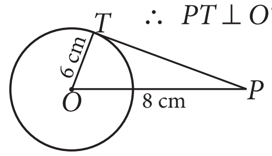
Also, sum of first $m$ terms, $S_m = \frac{m}{2}[2a+(m-1)d]$

$= \frac{m}{2}[2(5)+(m-1)2]$

$= m(5 + m - 1) = m(m + 4) = m^2 + 4m$

**13.** (i) Let $TP$ be the tangent to the circle and $O$ be the centre of circle.

$\angle OTP = 90°$ [∵ $PT$ is tangent to the circle ∴ $PT \perp OT$]




In $\Delta OTP$,

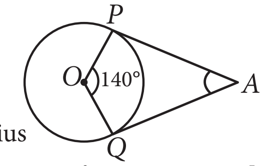
by Pythagoras theorem

$OP^2 = OT^2 + TP^2$

⇒ $8^2 = 6^2 + TP^2$

⇒ $TP^2 = 64 - 36 = 28 \Rightarrow TP = \sqrt{28} = 2\sqrt{7}$ cm

(ii) Let $PA$ and $QA$ be two tangents at the ends of radii $OP$ and $OQ$ respectively.




$\angle POQ = 140°$ [Given]

$\angle OPA = 90°$ and $\angle OQA = 90°$

[∵ Angle between tangent and radius through the point of contact is 90°]

In quadrilateral $OPAQ$,

$\angle POQ + \angle OPA + \angle OQA + \angle PAQ = 360°$

[Angle sum property of a quadrilateral]

⇒ $140° + 90° + 90° + \angle PAQ = 360°$

⇒ $\angle PAQ = 360° - 320° = 40°$

**14.** We have, radius of each coin = 3.5 cm

$= \frac{35}{10}\text{cm} = \frac{7}{2}\text{cm}$

Thickness of each coin = 0.5 cm $= \frac{1}{2}$cm

So, height of cylinder made by Meera$(h_1) = 12 \times \frac{1}{2} = 6$ cm

and height of cylinder made by Dhara $(h_2) = 8 \times \frac{1}{2} = 4$ cm

(i) Required ratio

$= \frac{\text{Curved surface area of cylinder made by Meera}}{\text{Curved surface area of cylinder made by Dhara}}$

$= \frac{2\pi r h_1}{2\pi r h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ *i.e.*, 3 : 2

(ii) Required ratio

$= \frac{\text{Volume of cylinder made by Meera}}{\text{Volume of cylinder made by Dhara}}$

$= \frac{\pi r^2 h_1}{\pi r^2 h_2} = \frac{h_1}{h_2} = \frac{6}{4} = \frac{3}{2}$ *i.e.*, 3 : 2